AP Environmental Science Math Prep

You are so lucky. Every other APES student before you has had to take an AP exam *without a calculator*. They suffered and groaned and begged. "Please, no more long division!". For the first time ever, the 2020 APES exam will allow calculators! So your summer math packet is shorter. Wave it in front of former APESters if you want to make them mad.

Common expectations for APES math:

1. **Write out all your work**, even if it's something really simple. This is required on the APES exam so it will be required on all your assignments, labs, quizzes, and tests as well.
2. **Include units** in each step. Your answers always need units and it's easier to keep track of them if you write them in every step.
3. **Check your work**. Go back through each step to make sure you didn't make any mistakes in your calculations. Check to see if your answer makes sense. For example, a person probably will not eat 13 million pounds of meat in a year. If you get an answer that seems unlikely, it probably is. Go back and check your work.

Directions

Read each section below for review. Look over the examples and use them for help on the practice problems. Stuck on a question? Bring it to class the first week of school.

Averaging

Skip this if you already know how to average. Don't feel bad if you don't.

To find an average, add all the quantities given and divide the total by the number of quantities.

Example: Find the average of 10, 20, 35, 45, and 105.

Step 1: Add all the quantities. 10 + 20 + 35 + 45 + 105 = 215

Step 2: Divide the total by the number of given quantities. 215 / 5 = 43

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

1. Find the average of the following numbers: 11, 12, 13, 14, 15, 23, and 29
2. Find the average of the following numbers: 124, 456, 788, and 343
3. Find the average of the following numbers: 4.56, .0078, 23.45, and .9872

Percentages

Percents show fractions or decimals with a denominator of 100. Always move the decimal TWO places to the right go from a decimal to a percentage or TWO places to the left to go from a percent to a decimal.

Examples: .85 = 85%. .008 = .8%

Part I: Finding the Percent of a Given Number

To find the percent of a given number, change the percent to a decimal and MULTIPLY.

Example: 30% of 400

Step 1: 30% = .30

Step 2: 400 x .30 = 120

Part II: Finding the Percentage of a Number

To find what percentage one number is of another, divide the first number by the second, then convert the decimal answer to a percentage.

Example: What percentage is 12 of 25?

Step 1: 12/25 = .48

Step 2: .48 x 100 = 48% (12 is 48% of 25)

Part III: Finding Percentage Increase or Decrease

To find a percentage increase or decrease, first find the percent change, then add or subtract the change to the original number.

Example: Kindles have dropped in price 18% from $139. What is the new price of a Kindle?

Step 1: $139 x .18 = $25

Step 2: $139 - $25 = $114

Part IV: Finding a Total Value

To find a total value, given a percentage of the value, DIVIDE the given number by the given percentage.

Example: If taxes on a new car are 8% and the taxes add up to $1600, how much is the new car?

Step 1: 8% = .08

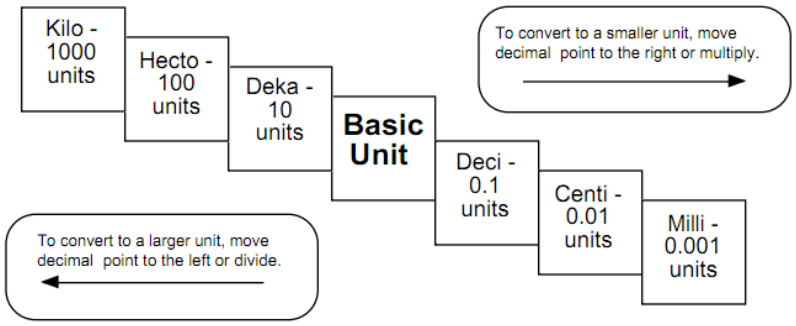
Step 2: $1600 / .08 = $160,000 / 8 = $20,000

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

4. What is 45% of 900?
5. Thirteen percent of a 12,000 acre forest is being logged. How many acres will be logged?
6. A water heater tank holds 280 gallons. Two percent of the water is lost as steam. How many gallons remain to be used?
7. What percentage is 25 of 162.5?
8. 35 is what percentage of 2800?
9. 14,000 acres of a 40,000 acre forest burned in a forest fire. What percentage of the forest was damaged?
10. Home prices have dropped 5% in the past three years. An average home in Indianapolis three years ago was $130,000. What's the average home price now?
11. The Greenland Ice Sheet contains 2,850,000 cubic kilometers of ice. It is melting at a rate of .006% per year. How many cubic kilometers are lost each year?
12. 235 acres, or 15%, of a forest is being logged. How large is the forest?
13. A teenager consumes 20% of her calories each day in the form of protein. If she is getting 700 calories a day from protein, how many calories is she consuming per day?

Metric Units

Kilo-, centi-, and milli- are the most frequently used prefixes of the metric system. You need to be able to go from one to another. You can remember the order of the prefixes by using the following sentence: *King Henry Died By Drinking Chocolate Milk*. Since the multiples and divisions of the base units are all factors of ten, you just need to move the decimal to convert from one to another.



Prefix	Prefix Symbol	Multiplier
mega-	**M**	**10^6 (1000000)**
kilo-	**k**	**10^3 (1000)**
BASE UNIT	**-**	**10^0 (1)**
centi-	**c**	**10^{-2} (0.01)**
milli-	**m**	**10^{-3} (0.001)**
micro-	**μ**	**10^{-6} (0.000001)**

Here's another chart with the most commonly used prefixes in APES:

Example: 55 centimeters = ? kilometers

Step 1: Figure out how many places to move the decimal. King Henry Died By Drinking… – that's six places. (Count the one you are going to, but not the one you are on.)

Step 2: Move the decimal five places to the left since you are going from smaller to larger.

55 centimeters = .00055 kilometers

Example: 19.5 kilograms = ? milligrams

Step 1: Figure out how many places to move the decimal. … Henry Died By Drinking Chocolate Milk – that's six places. (Remember to count the one you are going to, but not the one you are on.)

Step 2: Move the decimal six places to the right since you are going from larger to smaller. In this case you need to add zeros.

19.5 kilograms = 19,500,000 milligrams

Practice: Remember to show all your work, include units if given.

14. 1200 kilograms = ? milligrams
15. 14000 millimeters = ? meters
16. 6544 liters = ? milliliters
17. .078 kilometers = ? meters
18. 17 grams = ? kilograms

Scientific Notation

Introduction:

Scientific notation is a shorthand way to express large (anything over 1000) or tiny numbers. Writing these numbers in scientific notation will help you do your calculations much quicker and easier and will help prevent mistakes in conversions from one unit to another. Like the metric system, scientific notation is based on factors of 10. A large number written in scientific notation looks like this:

$$1.23 \times 10^{11}$$

The number before the x (1.23) is the **coefficient**, which must be greater than 1 and less than 10. The number after the x is the **base number** and is always 10. The number in superscript (11) is the **exponent**.

Part I: Writing Numbers in Scientific Notation

To write a large number in scientific notation, put a decimal after the first digit. Count the number of digits after the decimal you just wrote in. This will be the exponent. Drop any zeros so that the coefficient contains as few digits as possible.

Example: 123,000,000,000

Step 1: Place a decimal after the first digit. 1.23000000000

Step 2: Count the digits after the decimal…there are 11.

Step 3: Drop the zeros and write in the exponent. 1.23×10^{11}

Writing tiny numbers in scientific notation is similar. The only difference is the decimal is moved to the left and the exponent is a negative. A tiny number written in scientific notation looks like this:

$$4.26 \times 10^{-8}$$

To write a tiny number in scientific notation, move the decimal after the first digit that is not a zero. Count the number of digits before the decimal you just wrote in. This will be the exponent as a negative. Drop any zeros before or after the decimal.

Example: .0000000426

Step 1: 00000004.26

Step 2: Count the digits before the decimal…there are 8.

Step 3: Drop the zeros and write in the exponent as a negative. 4.26×10^{-8}

Part II: Adding and Subtracting Numbers in Scientific Notation

To add or subtract two numbers with exponents, the exponents must be the same. You can do this by moving the decimal one way or another to get the exponents the same. Once the exponents are the same, add (if it's an addition problem) or subtract (if it's a subtraction problem) the coefficients just as you would any regular addition problem (review the previous section about decimals if you need to). The exponent will stay the same. Make sure your answer has only one digit before the decimal – you may need to change the exponent of the answer.

Example: $1.35 \times 10^{6} + 3.72 \times 10^{5} = ?$

Step 1: Make sure both exponents are the same. It's usually easier to go with the larger exponent so you don't have to change the exponent in your answer, so let's make both exponents 6 for this problem.

$$3.72 \times 10^{5} \rightarrow .372 \times 10^{6}$$

Step 2: Add the coefficients just as you would regular decimals.

$$1.35 + .372 = 1.722$$

Step 3: Write your answer including the exponent, which is the same as what you started with.

$$1.722 \times 10^{6}$$

Part III: Multiplying and Dividing Numbers in Scientific Notation

To multiply exponents, multiply the coefficients just as you would regular decimals. Then add the exponents to each other. The exponents DO NOT have to be the same.

Example: $1.35 \times 10^{6} \quad X \quad 3.72 \times 10^{5} = ?$

Step 1: Multiply the coefficients.

$$(1.35)(3.72) = 5.022$$

Step 2: Add the exponents.

$$5 + 6 = 11$$

Step 3: *Write your final answer.*

$$5.022 \times 10^{11}$$

To divide exponents, divide the coefficients just as you would regular decimals, then subtract the exponents. In some cases, you may end up with a negative exponent.

Example: $5.635 \times 10^{3} \ / \ 2.45 \times 10^{6} = ?$

Step 1: Divide the coefficients.

$$5.635 / 3.45 = 2.3$$

Step 2: Subtract the exponents.

$$3 - 6 = -3$$

Step 3: Write your final answer.

$$2.3 \times 10^{-3}$$

Practice: Remember to show all your work, include units if given. All work and answers go on your answer sheet.

Write the following numbers in scientific notation:

19. 145,000,000,000
20. 13 million
21. .000348
22. 135 trillion
23. 24 thousand

Complete the following calculations:

24. $3 \times 10^{3} + 4 \times 10^{3}$
25. $7.89 \times 10^{-6} + 2.35 \times 10^{-8}$
26. $9.85 \times 10^{4} - 6.35 \times 10^{4}$
27. $2.9 \times 10^{11} - 3.7 \times 10^{13}$
28. three hundred thousand plus forty-seven thousand

29. 13 million minus 11 thousand
30. 1.32×10^8 X 2.34×10^4
31. three million times eighteen thousand
32. $3.45 \times 10^9 / 2.6 \times 10^3$
33. $1.98 \times 10^{-4} / 1.72 \times 10^{-6}$

Dimensional Analysis

Introduction

Dimensional analysis is a way to convert a quantity given in one unit to an equal quantity of another unit by lining up all the known values and multiplying. It is sometimes called factor-labeling. The best way to start a factor-labeling problem is by using what you already know. In some cases you may use more steps than a classmate to find the same answer, but it doesn't matter. Use what you know, even if the problem goes all the way across the page!

In a dimensional analysis problem, start with your given value and unit and then work toward your desired unit by writing equal values side by side. Remember you want to cancel each of the intermediate units. To cancel a unit on the top part of the problem, you have to get the unit on the bottom. Likewise, to cancel a unit that appears on the bottom part of the problem, you have to write it in on the top.

Once you have the problem written out, multiply across the top and bottom and then divide the top by the bottom.

Example: 3 years = ? seconds

Step 1: Start with the value and unit you are given. There may or may not be a number on the bottom.

$$\left[\frac{3 \text{ years}}{}\right]$$

Step 2: Start writing in all the values you know, making sure you can cancel top and bottom. Since you have years on top right now, you need to put years on the bottom in the next segment. Keep going, canceling units as you go, until you end up with the unit you want (in this case seconds)on the top.

$$\left[\frac{3 \text{ years}}{}\right] \left[\frac{365 \text{ days}}{1 \text{ year}}\right] \left[\frac{24 \text{ hours}}{1 \text{ day}}\right] \left[\frac{60 \text{ minutes}}{1 \text{ hour}}\right] \left[\frac{60 \text{ seconds}}{1 \text{ minute}}\right]$$

Step 3: Multiply all the values across the top. Write in scientific notation if it's a large number. Write units on your answer.

$$3 \times 365 \times 24 \times 60 \times 60 = 9.46 \times 10^7 \text{ seconds}$$

Step 4: Multiply all the values across the bottom. Write in scientific notation if it's a large number. Write units on your answer if there are any. In this case everything was cancelled so there are no units.

$$1 \times 1 \times 1 \times 1 = 1$$

Step 5: Divide the top number by the bottom number. Remember to include units.

$$9.46 \times 10^7 \text{ seconds} / \ 1 = 9.46 \times 10^7 \text{ seconds}$$

Step 6: Review your answer to see if it makes sense. 9.46×10^7 is a really big number. Does it make sense for there to be a lot of seconds in three years? YES! If you had gotten a tiny number, then you would need to go back and check for mistakes.

In lots of APES problems, you will need to convert both the top and bottom unit. Don't panic! Just convert the top one first and then the bottom.

Example: 50 miles per hour = ? feet per second

Step 1: Start with the value and units you are given. In this case there is a unit on top and on bottom.

$$\left[\frac{50 \text{ miles}}{1 \text{ hour}}\right]$$

Step 2: Convert miles to feet first.

$$\left[\frac{50 \text{ miles}}{1 \text{ hour}}\right] \left[\frac{5280 \text{ feet}}{1 \text{ mile}}\right]$$

Step 3: Continue the problem by converting hours to seconds.

$$\left[\frac{50 \text{ miles}}{1 \text{ hour}}\right] \left[\frac{5280 \text{ feet}}{1 \text{ mile}}\right] \left[\frac{1 \text{ hour}}{60 \text{ minutes}}\right] \left[\frac{1 \text{ minute}}{60 \text{ seconds}}\right]$$

Step 4: Multiply across the top and bottom. Divide the top by the bottom. Be sure to include units on each step. Use scientific notation for large numbers.

$$50 \times 5280 \text{ feet} \times 1 \times 1 = 264000 \text{ feet}$$
$$1 \times 1 \times 60 \times 60 \text{ seconds} = 3600 \text{ seconds}$$
$$264000 \text{ feet} / 3600 \text{ seconds} = 73.33 \text{ feet/second}$$

Practice: Remember to show all your work, include units if given. All work and answers go on your answer sheet. Use scientific notation when appropriate.

Conversions:

1 square mile = 640 acres	1 hectare (Ha) = 2.47 acres	1 kw-hr = 3,413 BTUs
1 barrel of oil = 159 liters	1 metric ton = 1000 kg	

34. 134 miles = ? inches
35. 8.9×10^5 tons = ? ounces
36. 1.35 kilometers per second = ? miles per hour
37. A city that uses ten billion BTUs of energy each month is using how many kilowatt-hours of energy?
38. A 340 million square mile forest is how many hectares?
61. If one barrel of crude oil provides six million BTUs of energy, how many BTUs of energy will one liter of crude oil provide?
62. Fifty eight thousand kilograms of solid waste is equivalent to how many metric tons?